

# Segmenting SAR Images using Fuzzy Clustering

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## Abstract

*Polarimetric Synthetic Aperture Radar (SAR) Images have great potential for land use management provided the images can be efficiently segmented. Clustering is one segmentation technique currently being explored. This paper compares two different fuzzy clustering techniques on SAR images that minimize two different objective functions. Examples of both methods are presented and future efforts to improve both results discussed.*

## 1. Introduction

JS Lee [1-2] has applied both hard c-means clustering (HCM) and fuzzy c-means clustering (FCM) to Synthetic Aperture Radar (SAR) images. Verdi et. al. [3] has also studied this approach on polarimetric high resolution tri-band SAR data and shown encouraging results for both the HCM and the FCM. This paper describes the continuation of this research and specifically addresses the reduction of the influence of outliers using robust fuzzy c-means (RFCM). A comparison of RFCM to the FCM using the Wishart distance measure is made. In section 2, the data space of polarimetric SAR Images is briefly described and previous work on segmenting SAR data with clustering is discussed. In section 3, a brief discussion of the versions of the FCM that apply to this problem is given, with emphasis on the RFCM. Section 4 gives some examples and section 5 contains the conclusions.

## 2. Polarimetric SAR Images

Polarimetric SAR images can be constructed from the complex scattering returns from the four possible polar combinations of transmit-receive returns of the radar: HH, HV, VH, and VV. Because

of symmetry assumptions, the HV and VH returns are identical yielding a 3-D complex scattering vector for each pixel in the image lattice. An incredible amount of preprocessing is required to form, register, and calibrate the image. The only feature used in this paper is the Coherence matrix, which is a Hermitian matrix defined as the outer product of three linear combinations of the complex scattering vectors: HH + VV, HV, and HH - VV. A real vector of dimension 9 is then constructed from the lower triangular part of this matrix. It is this feature vector that is associated with each pixel of the image lattice and used for clustering and classification. The dynamic range of this feature vector may be large and outliers are a frequent occurrence.

Du and Lee [2] applied FCM clustering to segment SAR images using a distance measure based on the Wishart Distribution. The form of the Wishart measure,  $d_{ik}^2 = \ln(|V_i|) + \text{tr}(V_i^{-1}X_k)$ , replaced the usual Euclidean squared distance  $d_{ik}^2$  in the FCM objective function. The RFCM is an alternative approach, that replaces  $d_{ik}^2$  in the objective function with Huber's  $\rho$  function. Both approaches reduce the influence of outliers by replacing  $d_{ik}^2$  by more slowly increasing function of distance.

## 3. Segmentation via Fuzzy Clustering

Clustering is often used to segment images since segmentation is really pattern recognition, i.e. classifying each pixel [4-5]. After clustering the pixel feature vectors into c-classes, one segments the image by labeling each pixel with the exemplar closest to it. The clustering method can employ either crisp sets as with the HCM or fuzzy sets as with the FCM and RFCM.

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### 3.1 Fuzzy c-means and Hard c-means

The HCM clustering algorithm is described in [6, p.55] and with the Wishart Measure in [1]. The FCM is a practical clustering algorithm that generalizes the HCM by replacing the class assignment with a membership vector whose elements represent the membership of the data point in each of the  $c$ -classes. The algorithm produces a fuzzy partition of the data and may be viewed as an unsupervised learning technique. The following description of the FCM is based on [6].

Consider  $N$  data samples forming the data set denoted by  $X = \{x_1, x_2, \dots, x_N\}$ , where each sample  $x_i \in R^p$ . Assume that there are  $c$  classes and  $u_{ik} = u_i(x_k) \in [0,1]$  is the membership of the  $k$ -th sample  $x_k$  in the  $i$ -th class  $v_i$ , where  $v = (v_1, v_2, \dots, v_c)$  is the set of exemplars or prototypes and  $U$  is the membership matrix. Each sample point

$x_k$  satisfies the constraint that  $\sum_{i=1}^c u_{ik} = 1$ . The FCM algorithm minimizes the function

$$J(U, v) = \sum_{k=1}^N \sum_{i=1}^c u_{ik}^{m_c} d_{ik}^2 \text{ where } d_{ik} = \|v_i - x_k\|_2$$

subject to the above constraint. The alternating optimization (AO) method is one technique to minimize  $J(U, v)$ . The power  $m_c$  of the membership is called the weighting exponent. A detailed version of this algorithm is given in [6, p.66]. HCM and FCM exemplars are linear statistics or weighted averages of the data points where the weights are scaled versions of the memberships. Unfortunately, linear statistics are known to be vulnerable to outliers [7]. HCM may also be viewed as a special case of FCM where the weighting exponent  $m_c$  is 1, and the data sample memberships in the classes are either 0 or 1. The HCM is also known to be more easily trapped in local minimum than the FCM.

### 3.2 Fuzzy c-Medians (FCMED) clustering

The FCMED is a more robust than the FCM since its objective function  $J(U, v)$  uses an  $\ell_1$  metric  $\|\bullet\|_1$ ,

$$\text{where } J(U, v) = \sum_{k=1}^N \sum_{i=1}^c u_{ik}^{m_c} d_{ik},$$

$$d_{ik} = \|v_i - x_k\|_1 = \sum_{j=1}^p |x_k(j) - v_i(j)|$$

and  $p$  is the dimension of the data space. In this case, the fuzzy median is the optimal centering statistic [8]. Although this algorithm is robust, it is not considered here because of the time complexity  $O(cpN \lg N)$  and its onerous space complexity  $O(N)$ , where  $N$  is the number of pixels in the image.

### 3.3. Robust Fuzzy c-Means clustering

Here we replace  $d_{ik}^2$  in the objective functional with Huber's  $\rho$  function. The objective function is

$$J(U, v) = \sum_{k=1}^N \sum_{i=1}^c u_{ik}^{m_c} \rho(d_{ik}) \text{ where } d_{ik} = \|v_i - x_k\|_2.$$

The  $\rho$  function applied in the examples of section 4

$$\text{is } \rho(x) = \begin{cases} \frac{1}{2}x^2, & \text{if } |x| \leq 1 \\ |x| - \frac{1}{2}, & \text{if } |x| > 1 \end{cases} \text{ whose form is quadratic}$$

when close to the exemplar, but linear when far from the exemplar. This particular  $\rho$  function is the one used by Huber in his early papers. The optimal memberships are then given by:

$$u_{ik} = 1 / \left[ \sum_{j=1}^c \left( \frac{\rho_{ik}}{\rho_{jk}} \right)^{1/(m_c-1)} \right]$$

The exemplars are computed by the weighted mean

$$\text{given by: } v_i = \sum_{k=1}^N u_{ik}^{m_c} w_{ik} x_{ik} / \sum_{k=1}^N u_{ik}^{m_c} w_{ik}, \text{ where the}$$

Huber weights  $w_{ik}$  are dependent upon  $d_{ik}$  [9]. These estimates for  $v_i$  are W-estimators or robust recursive estimators because the weights  $w_{ik}$  are functions of  $v_i$ . The weights have the form  $w(x) = \psi(x)/x$  where  $\psi(x) = \rho'(x)$ . In this case

$$w(x) = \begin{cases} 1, & |x| \leq 1 \\ 1/|x|, & |x| > 1 \end{cases}, \text{ which has the effect of}$$

gradually reducing the influence of the outliers. So,  $v_i$  is a weighted combination of the sample values where the weights depend on both the membership of the  $k$ -th sample in the  $i$ -th cluster  $u_{ik}$  and a spatial Huber weight function [7]. The advantage of this estimator is that its time complexity is  $O(N)$  and its space complexity is  $O(c)$  - a huge advantage for large images over the FCMED. The disadvantage of this approach is that the W-estimator should be iterated at each stage of the FCM, which of course would increase its time complexity by a factor proportional to the number of iterations. Another disadvantage is that a scaling constant is needed in

the Huber weight, requiring an auxiliary estimate of dispersion since the scale is usually a multiple of the dispersion. Here the auxiliary information is obtained from a robust estimator, the median absolute deviations about the median (MAD). The RFCM algorithm has the same algorithmic structure as the FCM, except the RFCM is more robust. Since the RFCM is non-linear in nature, it requires a better initialization for the exemplars. If an exemplar is too far removed from any data point, the membership of all the data points to this exemplar will be essentially zero, and the algorithm needs to deal with this special case to avoid underflow/overflow problems. In this paper, this problem is avoided by using Huber weights, which have infinite support yet vanishing weight.

#### 4. Examples

In this section, several examples using various segmentation algorithms are given. Figure 1 is a 320x320 pixel POL-SAR span image where the total power in the HH, HV, and VV returns is properly censored and scaled. The vertical line is an image artifact whose cause is unknown. The segmentation algorithms are applied to this image.



Figure 1. POL-SAR big picture.

Figure 2 is the HCM clustering algorithm with the Wishart measure applied to the Coherence matrix that is constructed using a 3x3 moving average window with overlap. The initial nine cluster classes and their centers were computed using Cloude's alpha-entropy classification [10]. The 9 clusters are shown in 9 different levels. The building and the pond are clearly delineated and the trees that surround the pond are accurate since this area has been ground truth'd by one of the authors. The HCM was run for 10 iterations.

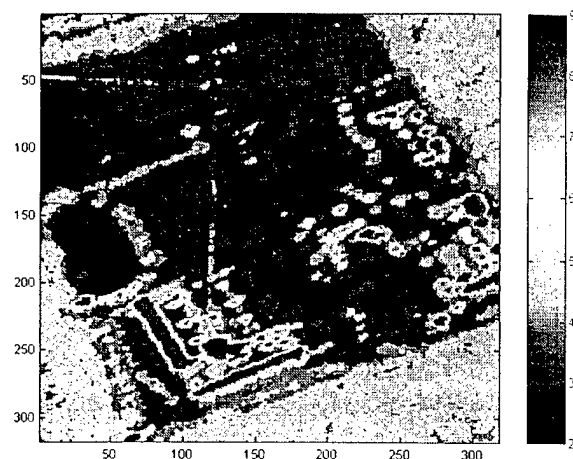


Figure 2. HCM with Wishart Distance, 10 iterations.

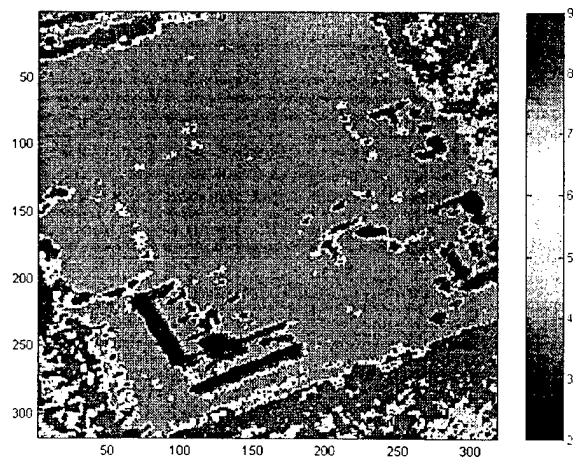
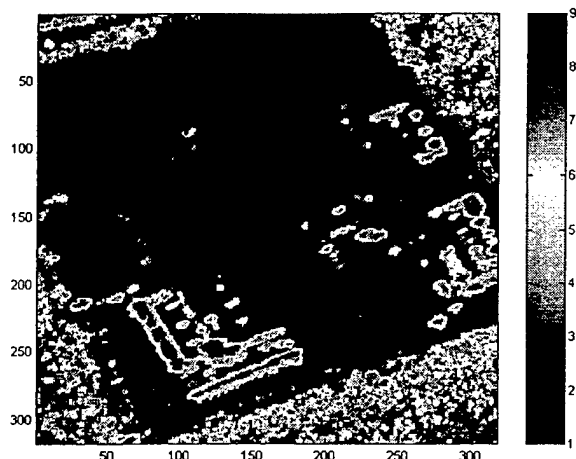


Figure 3. FCM with Wishart Measure, 10 iterations.

Figure 3 shows the results of processing the same data using the FCM with the weighting exponent  $m_c = 1.125$ . This segmentation defines the building better and tends to treat the water, grass and parking lot the same - i.e. relatively flat smooth surfaces are colored the same. The RFCM for this image using the Wishart measure is virtually identical, which indicates that the image has already obtained its resistance to outliers from the distance measure before the robustized algorithm can impact the process.

If the Euclidean metric is used instead of the Wishart measure, then the FCM segmentation produces disastrous results. In contrast, the RFCM produces results comparable to the Wishart measure and the FCM.



**Figure 4. RFCM with Euclidean Metric, 10 iterations.**

Using the same initial vectors and the Huber  $\rho$  function, with an appropriate scaling and tuning constant, the RFCM results are shown in figure 4. Similar results can also be achieved using the Tukey Biweight, but the same initialization vectors cannot be used because the Biweight has finite support and some of the initialization vectors are too far away from the data, which causes numerical problems. The infinite support of the Huber weights allows the RFCM to better search for a cluster, if the initialization vector is not near a cluster. However, the Tukey Biweight can only search for clusters over a finite range since it has only finite support. Of course, if a good initial estimate is known, the Tukey Biweight is a better choice. The RFCM does not require a sophisticated initialization to work as observed in the above examples where the exemplar initialization is far from the final fuzzy cluster centers.

## 5. Conclusions

The large dynamic range of the POL-SAR data demands a robust version of the FCM clustering algorithm. Two examples of such clustering techniques have been given. The first obtains its resistance to outliers from the Wishart measure and the second obtains its resistance via Huber's function. Both methods soften the squared error term to reduce the influence of outliers. Our initial research shows both of these approaches are viable and a detailed comparison is the subject of future research. Because of the nonlinear character of the RFCM, a judicious choice of the initial exemplars can accelerate the convergence. The proper number of clusters is needed in both algorithms to efficiently segment the

image with a lower probability of error. Our research continues in this area.

## 6. References

1. J. S. Lee, M. R. Grunes, T. L. Ainsworth, L. Du, D. L. Schuler and S. R. Cloude, "Unsupervised Classification Using Polarimetric Decomposition and Complex Wishart Classifier," Proceedings IGARSS 98, Seattle, WA, pp. 2178-2180, 1998 July 6-10.
2. L. Du and J.S. Lee, "Fuzzy classification of earth terrain covers using complex polarimetric SAR data, Int. J. Remote Sensing, 1996, Vol. 17, No 4, pp. 809-926.
3. J. Verdi, R. Lee, P. R. Kersten, et. al, "Comparative Analysis of Polarimetric SAR Image Exploitation Algorithms for Environmental Stress-Change Monitoring," IGARSS Proceedings, Vol. V., 28 June - 2 July 1999, pp 2449-2451.
4. J. Bezdek and S. Pal, Fuzzy Models for Pattern Recognition, Bezdek and Pal Editors, IEEE Press, NY, 1992
5. J.C. Bezdek and M. M. Trivedi, Low level segmentation of aerial images with fuzzy clustering, IEEE Trans. Syst. Man. Cyb. SMC-16, 589-598 (1986). S. R. Cloude, E. Pottier, "An Entropy Based Classification Scheme for Land Applications of Polarimetric SAR," IEEE Trans. GRS, Vol. 35, No. 1, pp. 68-78, January 1997.
6. J. Bezdek, Pattern Recognition with Fuzzy Objective Function Algorithms, Plenum Press, New York, 1981
7. D.C. Hoagen, F. Mosteller, and J.W. Tukey, *Understanding Robust and Exploratory Data Analysis*, Wiley, New York, 1983, p. 367.
8. Kersten, P., "Fuzzy Order Statistics and Its Application to Fuzzy Clustering, IEEE Transactions on Fuzzy Systems, Vol 7, No 6, Dec 1999, pp 708-712.
9. Y Choi and R. Krishnapuram, "Fuzzy and Robust Formulations of Maximum-Likelihood-Based Gaussian Mixture Decomposition," Fifth IEEE International Conference on Fuzzy Systems, Sept 8-11, New Orleans, LA, Vol 3, p. 1899-1905.
10. S. R. Cloude, E. Pottier, "An Entropy Based Classification Scheme for Land Applications of Polarimetric SAR," IEEE Trans. GRS, Vol. 35, No. 1, pp. 68-78, January 1997.

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